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Technical Notes

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Uncertainty Analysis of LE-7A Liquid-Oxygen-Hydrogen Rocket-Engine Hot Firing Tests

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Nomenclature

B_r = systematic uncertainty calculated parameter
F_{vac} = vacuum thrust
I_{vac} = vacuum specific impulse
S_X = standard deviation
U = total uncertainty

Introduction

THE H-IIA rocket is the primary large-scale launch vehicle of the Japan Aerospace Exploration Agency (JAXA). The vehicle is primarily used for launching satellite payloads and injecting them into orbit. The first-stage main engine for the H-IIA is the LE-7A engine. The LE-7A is a liquid-oxygen/hydrogen engine with a staged combustion power cycle. Development of the LE-7A engine began in 1994 as part of a larger redesign of the H-II rocket. The engine was developed to replace the LE-7 first-stage engine of the H-II rocket. The LE-7A was designed to improve several aspects of its predecessor including reliability, performance, and operational costs.[1]

Each LE-7A engine is subjected to a series of acceptance tests to verify that it meets performance requirements. In the LE-7A acceptance test, the engine is tested at a steady set point for 50 s, and the measured values of the variables (thrust, flow rates, etc.) are recorded as averages over 1-s intervals. The data used in this analysis are the measurements from the time period 10 to 50 s in these tests.

Data from acceptance tests for six individual LE-7A engines were considered as part of this research program. The data consisted of three independent firings from each of the six engines. It was assumed that the third test for each engine met the criteria established for engine operation. Using that assumption, any consecutive test of the same engine with the same configuration as the third test was also assumed to meet the established criteria. In general, the first test for each engine was eliminated from the data sets, while the second and third tests were kept in the data set. After eliminating this data, 11 tests from six different engines remained. This set of data was used as the basis for this study.

A detailed uncertainty analysis was performed for the test firings of the LE-7A engine. Uncertainty sources associated with the test firings were evaluated. Total uncertainties for each test, each engine, and for data-set average values were determined and are presented in this work. The uncertainty analysis focuses on two standard performance equations: vacuum thrust F_{vac} and vacuum specific impulse I_{vac}. Details on the derivation of these equations can be found in Ref. 2.

Uncertainty Equations

In this analysis, it is assumed that the measured variables are considered to be at a steady state during the time period of interest. It is also assumed that any error whose value is known has been eliminated by applying a correction. The uncertainty methodology outlined in Ref. 3 was used in the analysis with a few exceptions, noted in the following.

It was discovered that the random uncertainty in the measured variables contained correlated random errors caused by test unsteadiness. That is, the fluctuations of one variable with respect to another were not completely random. Accounting for this effect using standard propagation techniques is difficult. However the random uncertainty of a result can be calculated directly by constructing a result for each set of measurements, that is, a result at 10, 11, 12 s, etc. The random uncertainty in the result is then determined as twice the standard deviation of the result data set. This method automatically includes the correlated random uncertainty values. Reference 4 discusses this point in more detail.

In many cases, the uncertainty of the mean $\bar{U}_r$ is not as useful as the uncertainty U_r. Both uncertainties are presented; however, the uncertainty U_r is more valuable for interpreting the results in this case. This point and detailed information on the uncertainty analysis are presented in Ref. 4.

The significant systematic uncertainties have been summarized in Table 1. The uncertainties were based on multiple sources: experimental data, calibration, and estimated from reference sources. Reference 4 provides information on how these uncertainties were determined.

The variations of measured data while at a presumed steady state represent a source of random uncertainty associated with the measured quantity. Table 2 provides the random uncertainty associated with vacuum thrust and vacuum specific impulse. Both the random uncertainty and random uncertainty of the mean are provided in the table.

Results

Total uncertainty values were determined for the performance parameters using random uncertainty $2 \cdot S_r$, which characterizes parameter variation over the test period, random uncertainty $2 \cdot \bar{S}_r$ for test average values, and a random uncertainty estimate appropriate

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Table 1 Systematic uncertainty estimates

Quantity	B	Basis
F1	900 N	Analysis
F2	900 N	Analysis
QF	0.17%	JAXA data
QO	0.15%	JAXA data
ρ_{LOX}	0.25%	Ref. 4
ρ_{LH2}	0.25%	Ref. 4
Dia	1.22 mm	Analysis
Patm	0.05 kPa	JAXA data

Table 2 Random uncertainty values (% of average)

	F_{vac}		Isp	
Test number	$2\,Sx$	$2\,Sx$	$2\,Sx$	$2\,Sx$
1	1.20	0.13	0.80	0.19
2	1.10	0.07	0.40	0.17
3	0.70	0.12	0.80	0.11
4	1.10	0.09	0.60	0.17
5	0.60	0.07	0.50	0.09
6	0.40	0.06	0.40	0.06
7	1.40	0.09	0.60	0.22
8	0.90	0.15	1.00	0.14
9	1.30	0.08	0.50	0.21
10	1.10	0.14	0.90	0.17
11	1.60	0.08	0.50	0.26

Table 3 Test uncertainties U_r and $\bar{U}_r$ (% of average)

	Single test uncertainty		Test average uncertainty	
Test	U_{Fvac}	U_{Ivac}	U_{Fvac}	U_{Ivac}
1	1.19	0.85	0.20	0.29
2	1.10	0.50	0.18	0.27
3	0.70	0.84	0.13	0.29
4	1.09	0.60	0.18	0.28
5	0.59	0.53	0.11	0.27
6	0.42	0.47	0.09	0.27
7	1.41	0.62	0.23	0.28
8	0.91	1.00	0.16	0.30
9	1.32	0.60	0.22	0.27
10	1.10	0.95	0.18	0.30
11	1.64	0.57	0.26	0.27

for a total data set average (average of all tests in the final data set). When consecutive tests were performed on the same engine, with no change to the configuration, the tests were compared (as discussed next) to provide an estimate of the repeatability of the test average magnitudes. The results from these analyses are presented in the following.

Single Test Uncertainty U_r

Systematic and random uncertainties for a set of results calculated from measurements at specific times during a test were provided in preceding sections. This type of analysis produces an uncertainty band $\pm U_r$ within which the "true" value of the calculated quantity falls at any instant within the test with 95% confidence. For example, for engine 1 the analysis of test 1 indicates that at any instant during the test the true thrust that the engine produces lies within $\pm 1.19\%$ of the test average value. The single test uncertainty values are provided in Table 3.

Test-Average Value Uncertainty $\bar{U}_r$

When looking at an average value from a single test, in random-uncertainty-dominated estimates the total uncertainty $\bar{U}_r$ is much smaller in magnitude than U_r because the contribution of random uncertainty diminishes. The uncertainty associated with an average value represents the band in which the true average of the data during the averaging time period will lie with 95% confidence. For example, the true average value of thrust for test 1 will be within $\pm 0.20\%$ of the data-set average value. This uncertainty band might not be very useful in tests such as those considered here. When the variation of the measured values (the random uncertainty) is a dominant uncertainty source, it is important to have the steady-state process operate for a long time span, so that the range of the variation can be seen. In short tests, the variation of a single test might not be representative of the variation of the process over a longer time span. Values for the total uncertainty $\bar{U}_r$ of the test average values are also provided in Table 3.

Engine-Average Values

Tests on the same engine were compared to examine repeatability for the same engine. Values of vacuum thrust and vacuum specific impulse were useful for this comparison because these values are independent of the test-to-test variation in atmospheric pressure. Figure 1 shows vacuum thrust average values $\bar{F}_{\mathrm{vac}}$ with the U_r uncertainty values for each test in the data set. Values from tests on the same engine are represented by the same symbols. In Fig. 1 all but one of the test-average values for the same engine lie within uncertainty bands of each other, with the test 11 result barely being outside the uncertainty band of test 10. This is a good indication that the majority of the significant random uncertainties (and variations within a test) has been captured in the analysis.

Figure 2 shows similar plots for the test-average values of vacuum specific impulse $\bar{I}_{\mathrm{vac}}$ for each test in the data set. In this figure values from tests on the same engine are within the uncertainty bands of each other. $\bar{U}_{\mathrm{Fvac}}$ and $\bar{U}_{\mathrm{Ivac}}$ uncertainties are not provided here because these values are not a good indication of the true variation from test to test. These values are provided in Ref. 4.

Eleven-Test-Averaged Values and Uncertainties

A final uncertainty analysis was performed for the collection of all 11 tests on the six different LE-7A engines in the data set. The individual test-average values of vacuum thrust and vacuum specific impulse were combined to produce 11-test-averaged values of $\bar{F}_{\mathrm{vac\_11}}$ and $\bar{I}_{\mathrm{vac\_11}}$. Figure 3 shows the vacuum thrust test average and U_{Fvac} for each of the 11 tests in the established data set. Also shown in the figure are the 11-test-averaged value $\bar{F}_{\mathrm{vac\_11}}$ and the

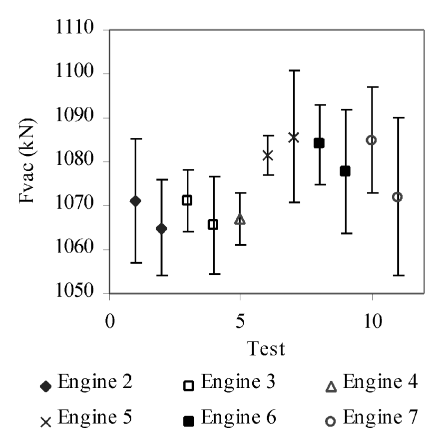

Fig. 1 F_{vac} with total uncertainty.

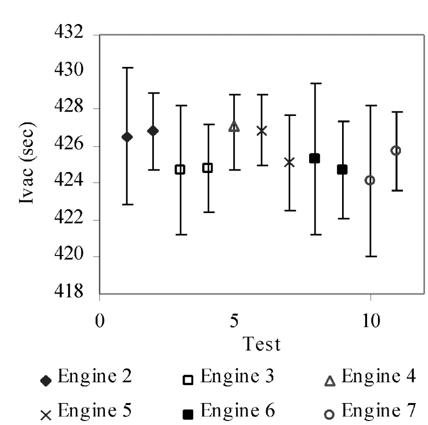

Fig. 2 I_{vac} with total uncertainty.

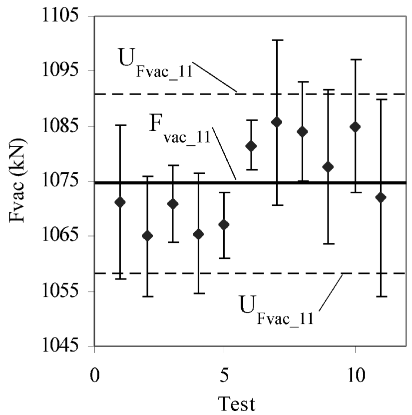

Fig. 3 Vacuum thrust average values $\bar{F}_{vac}$ with 11-test-averaged F_{vac_11}.

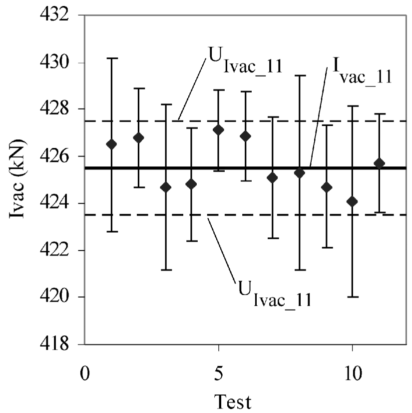

Fig. 4 Vacuum specific impulse average values $\bar{I}_{vac}$ with 11-test-averaged I_{vac_11}.

uncertainty band $\pm U_{Fvac\_11}$. Figure 4 shows similar information for vacuum specific impulse values from the 11 tests.

For vacuum thrust, the 11 test-averaged values were calculated as 1075 kN ± 16 kN. For vacuum specific impulse, the 11 test-averaged values were determined to be 426 s ± 2 s. These values are the ranges in which test-average values from all of the tests in the established data set fall. That is, all of the test-average values of vacuum thrust are included in the range 1075 kN ± 16 kN. These values closely represent the ranges in which values determined from additional tests of identical engines at the same operating conditions at the same test site would be expected to fall with 95% confidence. The values for $F_{vac\_11} \pm U_{Fvac\_11}$ are very close to the criteria used to determine if the engine passes the acceptance test.

Summary

An uncertainty analysis for vacuum thrust and vacuum specific impulse of the LE-7A liquid rocket engine has been performed using measured data from 11 hot firing tests. For a given test, the data were analyzed using a time period from 10 to 50 s after ignition.

Uncertainties $\bar{U}_r$ for the average value of a performance parameter during a test period were calculated. The interval $\pm\bar{U}_{Fvac1}$ centered on the average value of $\bar{F}_{vac}$ indicates the range within which the true value of $\bar{F}_{vac}$ during that particular test period falls with 95% confidence. For all performance parameters, the average value uncertainties $\bar{U}_r$ were between 0.09 to 0.52% of the average value. The $\bar{U}_r$ values are only useful when considering a single average during a specific time period. These uncertainties did not capture the test to test variation.

Uncertainty percentage contribution values indicated that the uncertainties U_r of the performance parameters were dominated by the random uncertainty of the tests. For all of the uncertainties U_r, the random uncertainty component accounted for at least 80% of the uncertainty squared in vacuum thrust and at least 48% of the uncertainty squared in vacuum specific impulse.[4]

The small influence of systematic uncertainty in the U_r estimates indicates that little to no advantage would be obtained by using resources to attempt to reduce the systematic uncertainties of the variables measured in the tests. For example, if the two largest systematic uncertainties in vacuum specific impulse were eliminated, the total uncertainty U_{Ivac} for any test in the data set would be reduced by less than 0.3 s, or 0.07% of the average value.

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